The Cordiality Game and the Game Cordiality Number

Elliot Krop, Aryan Mittal, and Michael Wigal

Abstract. The *cordiality game* is played on a graph G by two players, Admirable (A) and Impish (I), who take turns selecting vertices of G. Admirable labels the selected vertices by 0 and Impish by 1, and the resulting label on any edge is the sum modulo 2 of the labels of the vertices incident to that edge. The two players have opposite goals: Admirable attempts to minimize the number of edges with different labels as much as possible while Impish attempts to maximize this number. When both Admirable and Impish play their optimal games, we define the *game cordiality number*, $c_g(G)$, as the absolute difference between the number of edges labeled zero and one. Let P_n be the path on n vertices. We show $c_g(P_n) \leq \frac{n-3}{3}$ when $n \equiv 0 \pmod 3$, $c_g(P_n) \leq \frac{n-1}{3}$ when $n \equiv 1 \pmod 3$, and $c_g(P_n) \leq \frac{n+1}{3}$ when $n \equiv 2 \pmod 3$. Furthermore, we show a similar bound, $c_g(T)\frac{1}{2}n$ holds for any tree T. We conjecture that $c_g(T) \leq \frac{|T|+1}{3}$ for any tree T, and that $c_g(P_n)$ attains the values 0 and 1 for infinitely many integers n. We also show that the cordiality game is p-space complete and state some further directions of inquiry.

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Department of Mathematics, Clayton State University